Randomized fluid dynamics based on subgrid transport

Valentin Resseguier, Etienne Mémin, Bertrand Chapron

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Motivations

- Rigorously identified sudgrid dynamics effects
- Injecting likely small-scale dynamics
- Predicting possible distinct scenarios
- Quantification of modeling errors:
  - Diagnose to design numerical simulations (mesh refinements, …)
  - **Data assimilation: ensemble forecasts**

Contents

- Randomized dynamics

- SQG under Moderate Uncertainty

Randomized dynamics

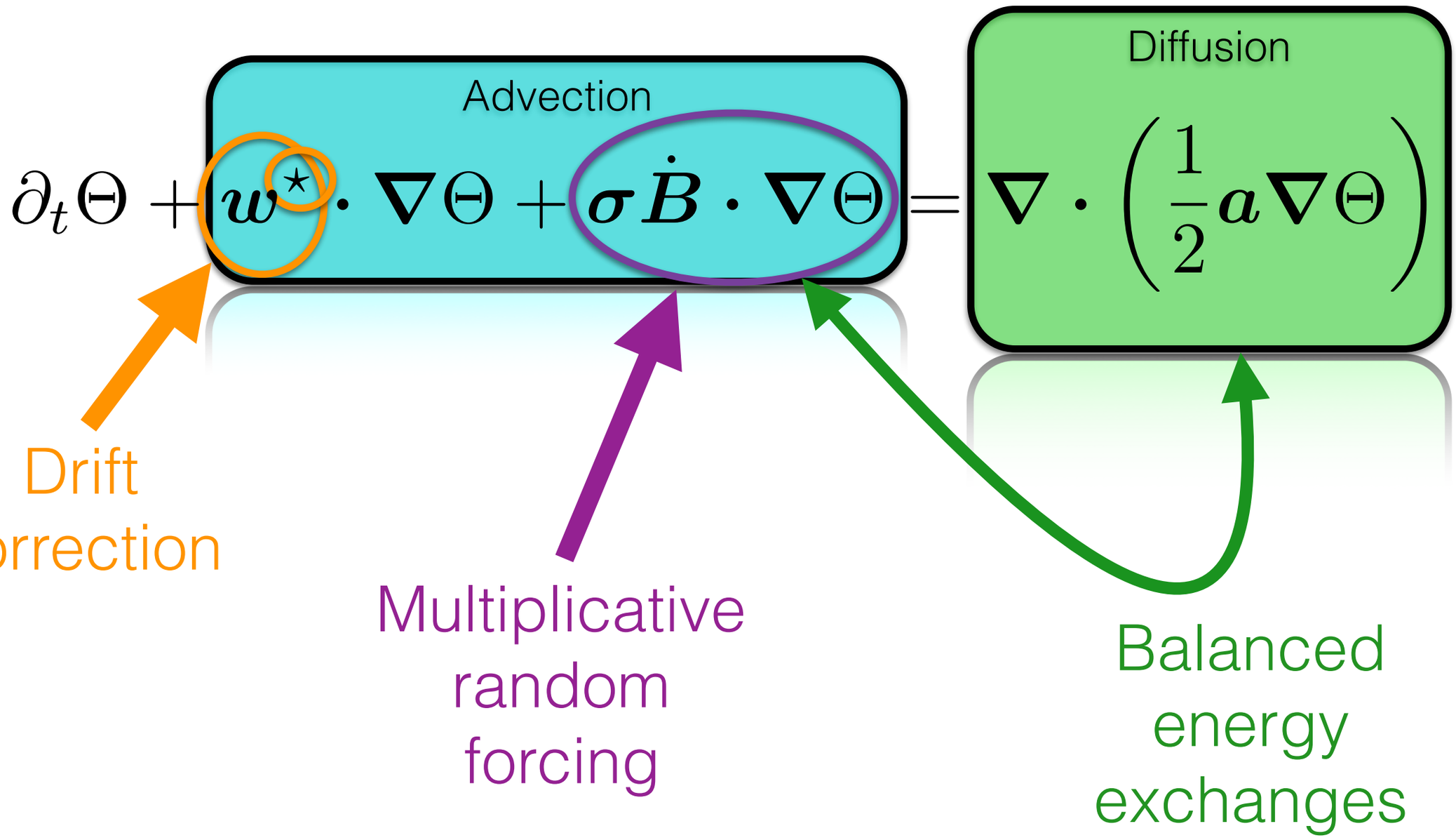
Random equations

- Random initial conditions → Underdispersive
- Arbitrary Gaussian forcing → Adding energy + wrong phase
- Averaging, homogenization → Previous talk
- Adding white random velocity → $\boldsymbol{v} = \boldsymbol{w} + \boldsymbol{\sigma}\dot{\boldsymbol{B}}$

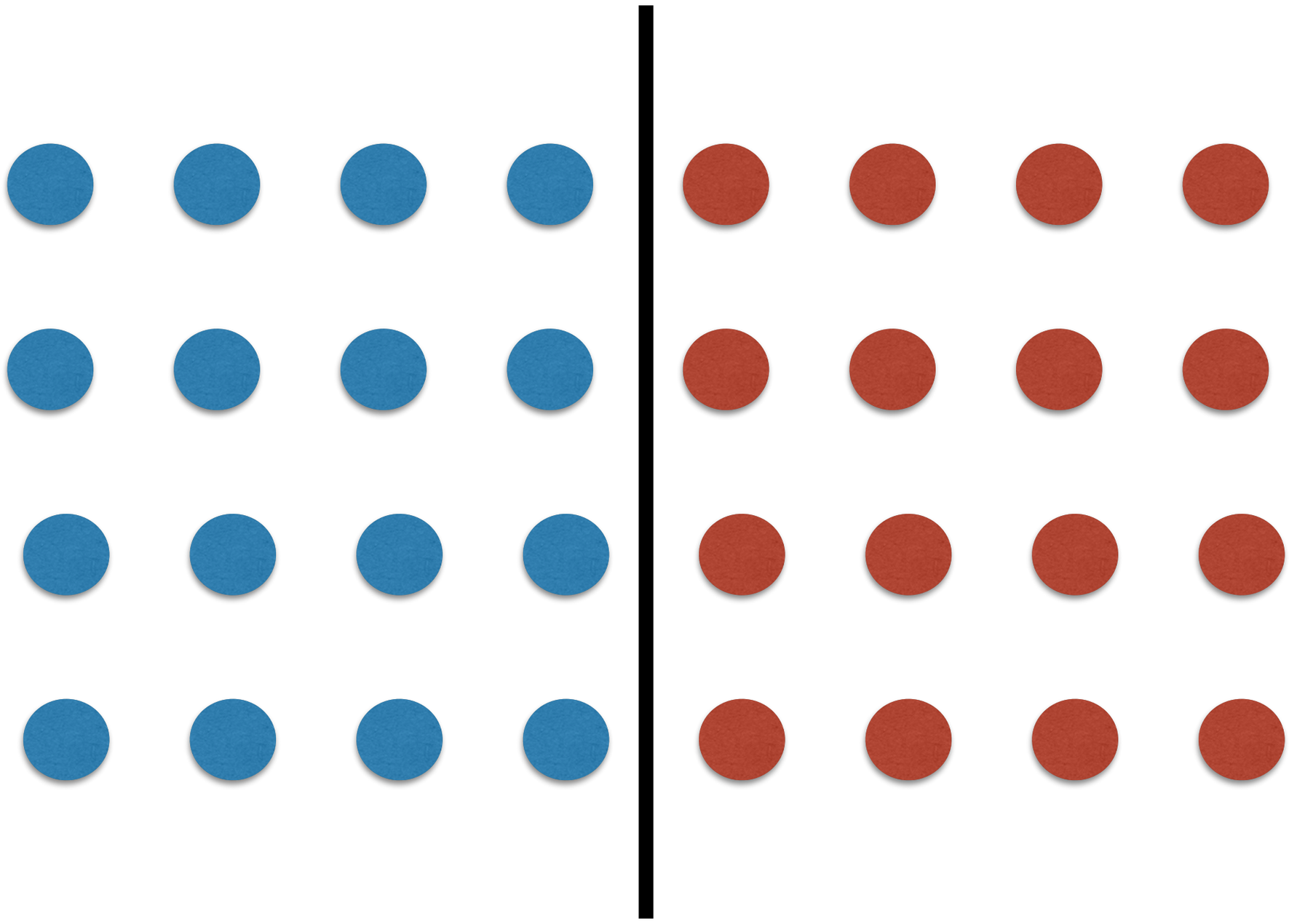
Advection of tracer Θ

$$\frac{D\Theta}{Dt} = 0$$

Advection of tracer Θ

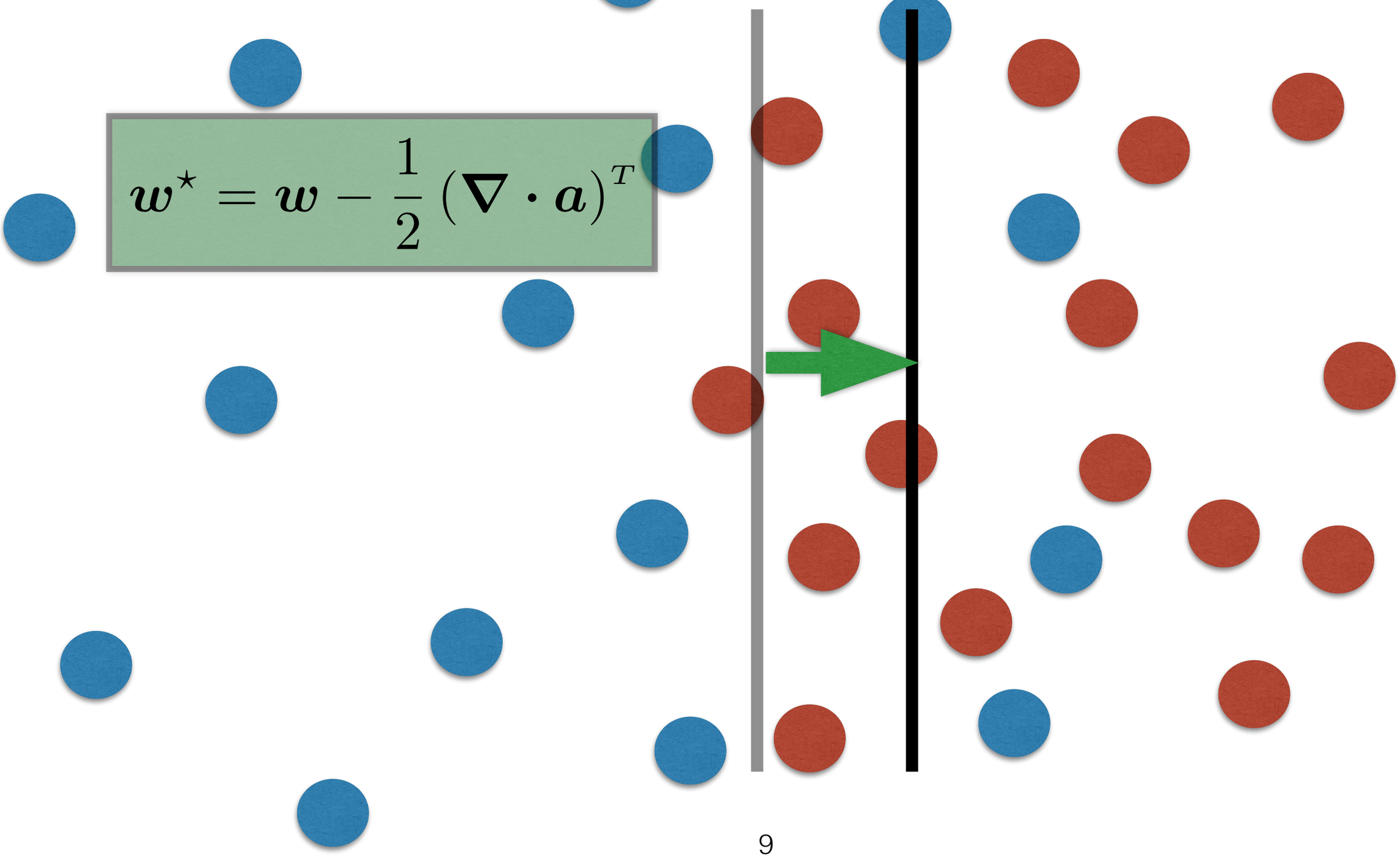


Drift correction

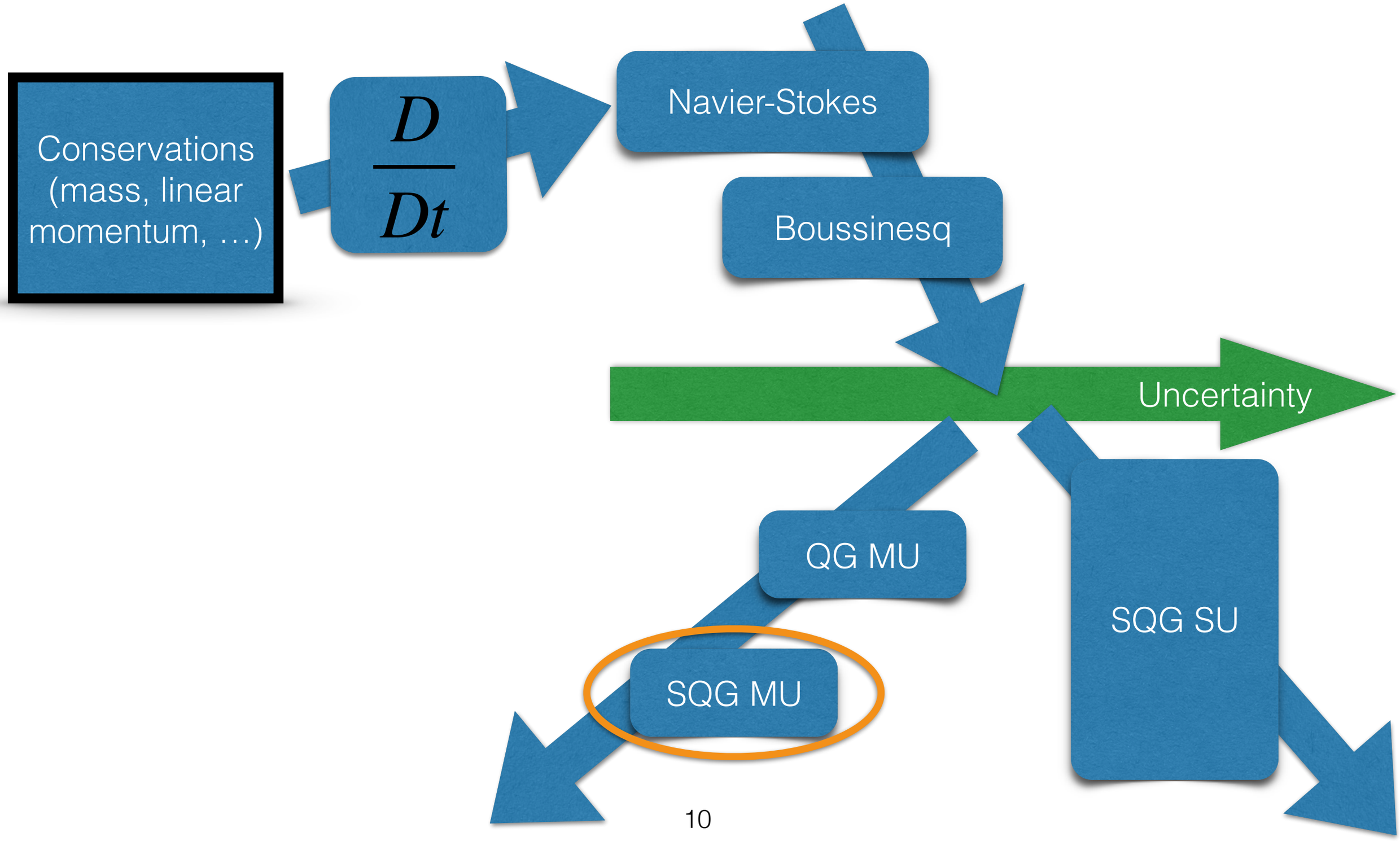


Drift correction

$$\boldsymbol{w}^{\star} = \boldsymbol{w} - \frac{1}{2}\left(\boldsymbol{\nabla} \cdot \boldsymbol{a}\right)^{T}$$



Derived random models



SQG under Moderate Uncertainty

SQG MU

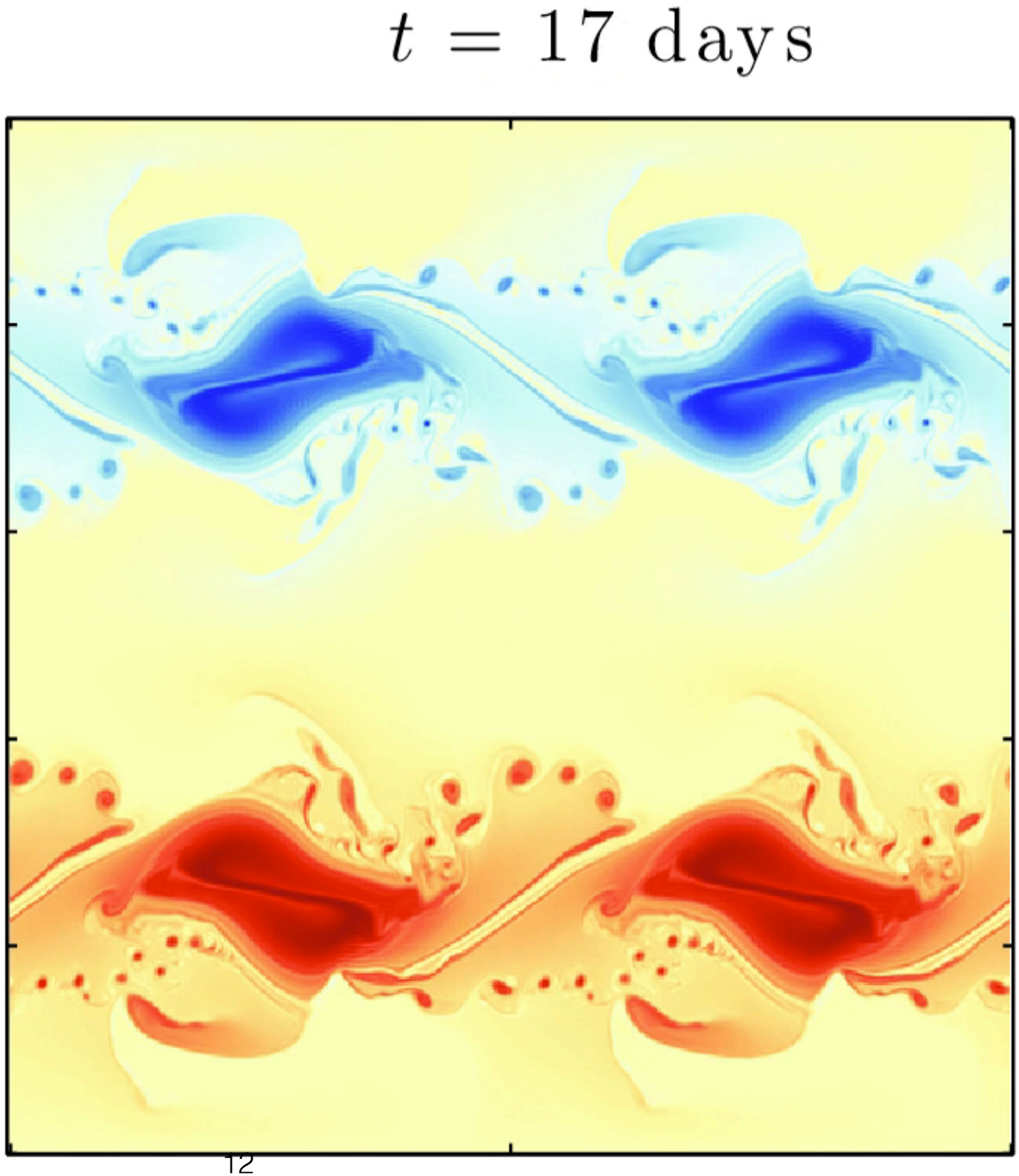
Code available online

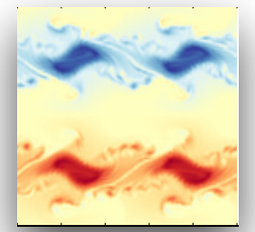
Reference flow:

deterministic

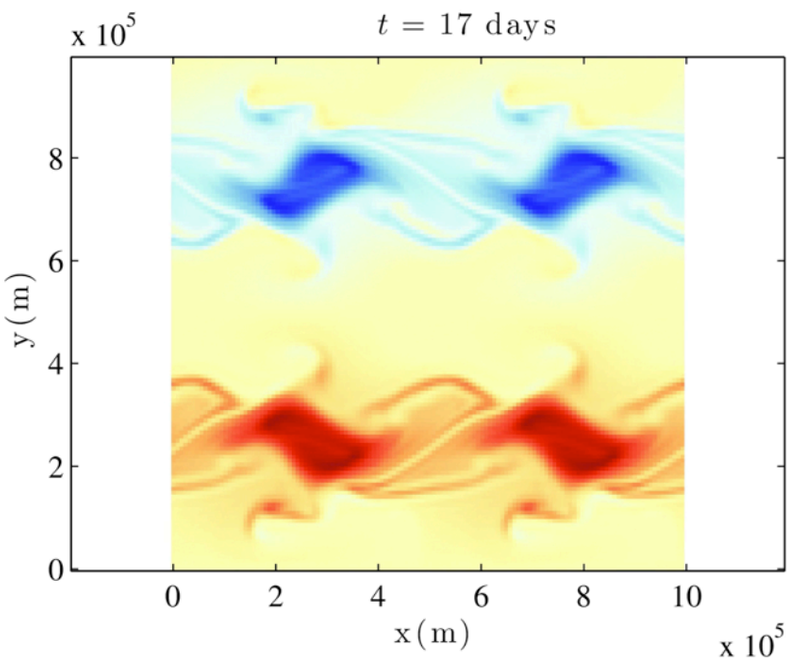
SQG

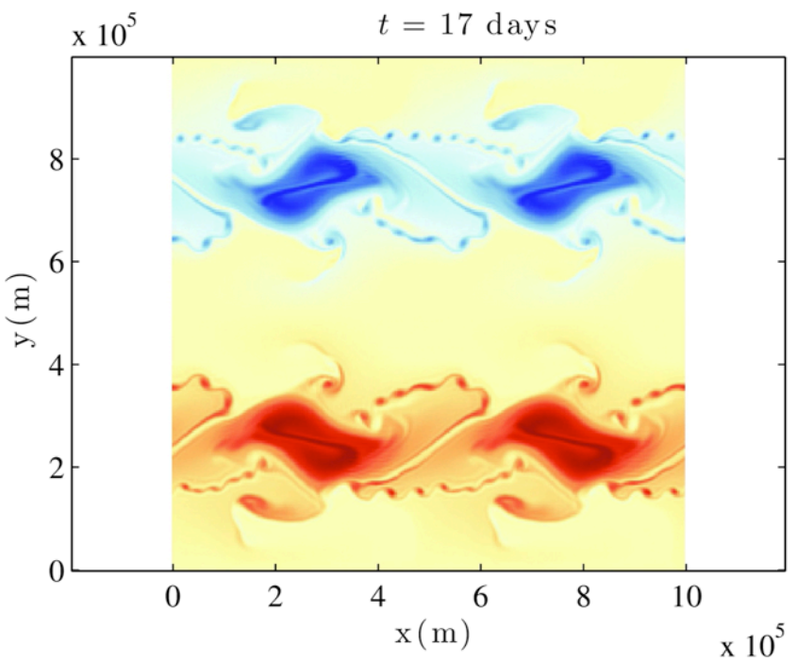
512 x 512

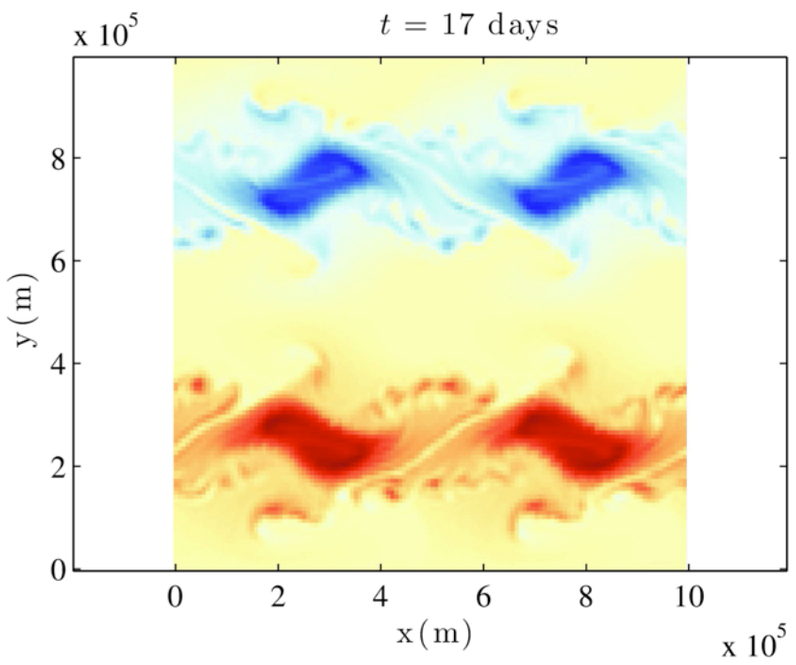


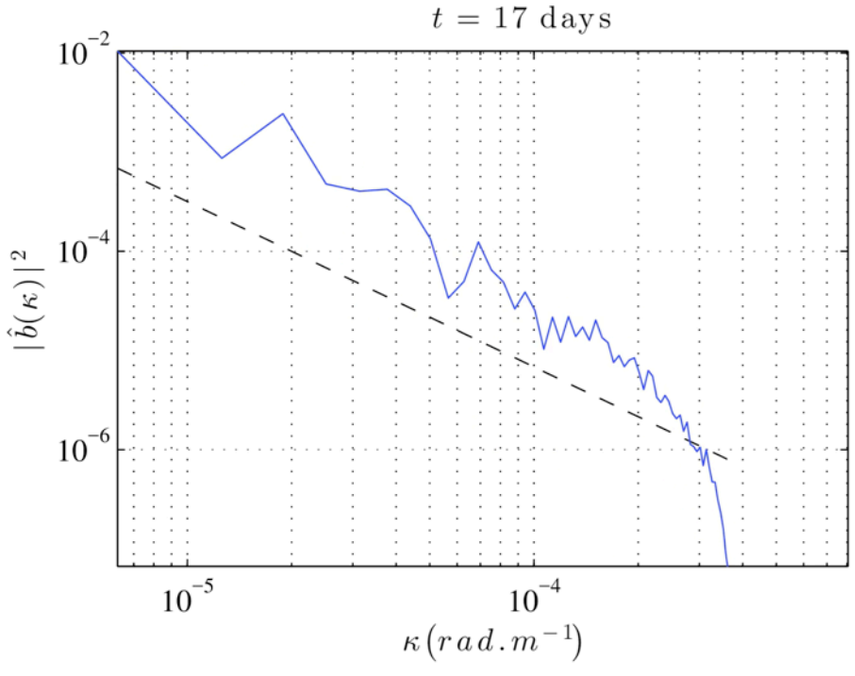


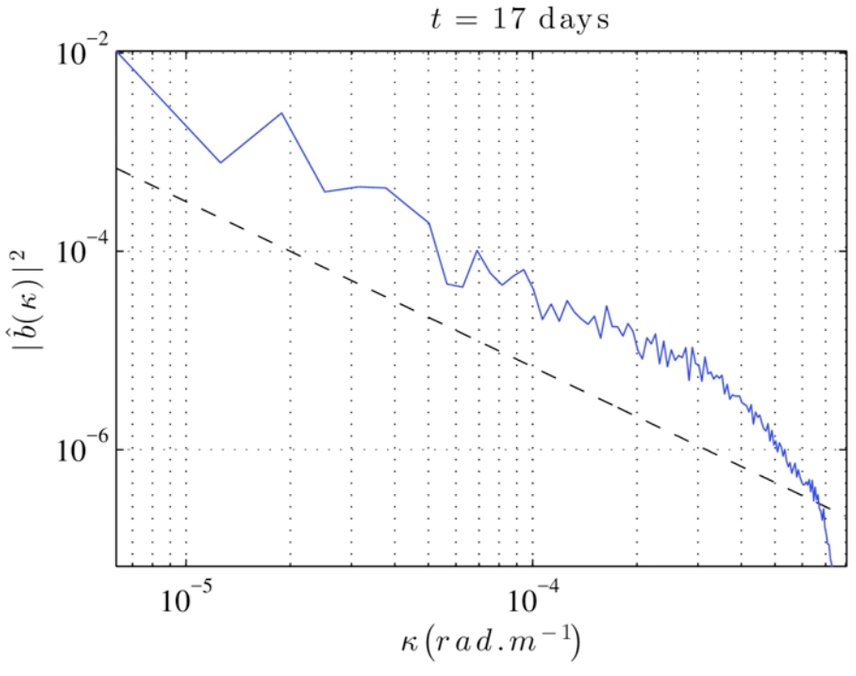
One realization

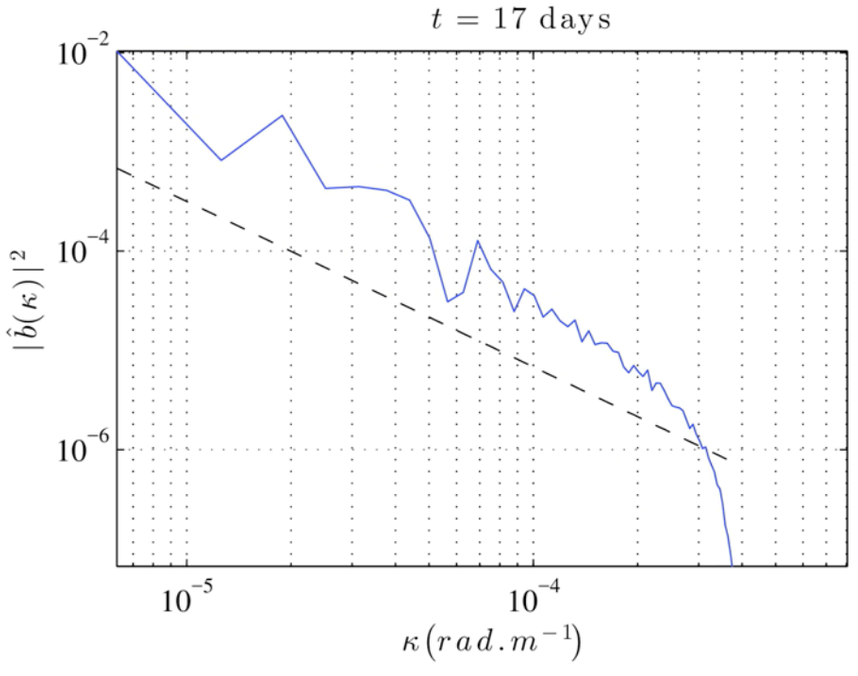








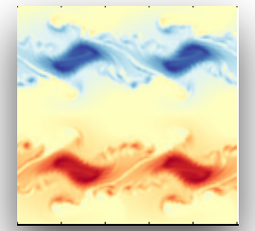




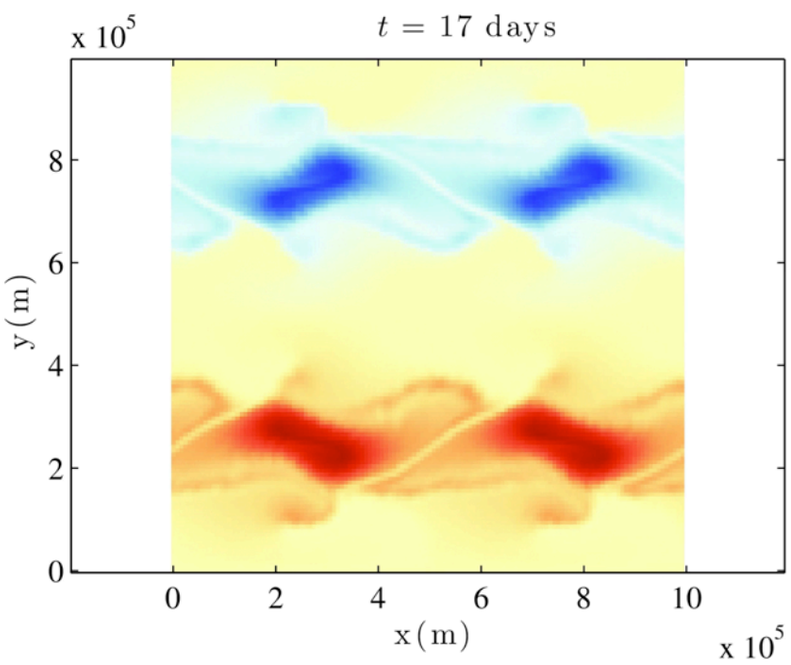
Deterministic 128x128

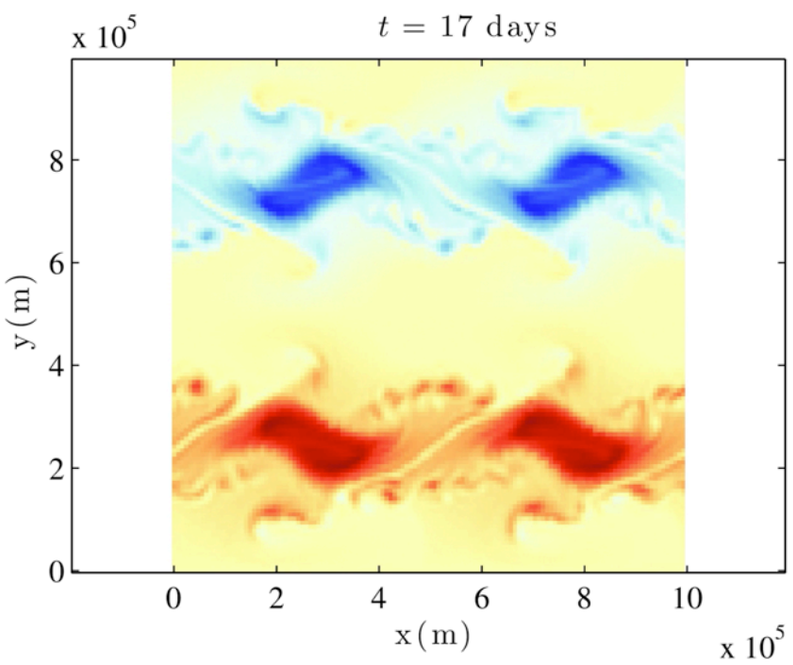
Deterministic 512x512

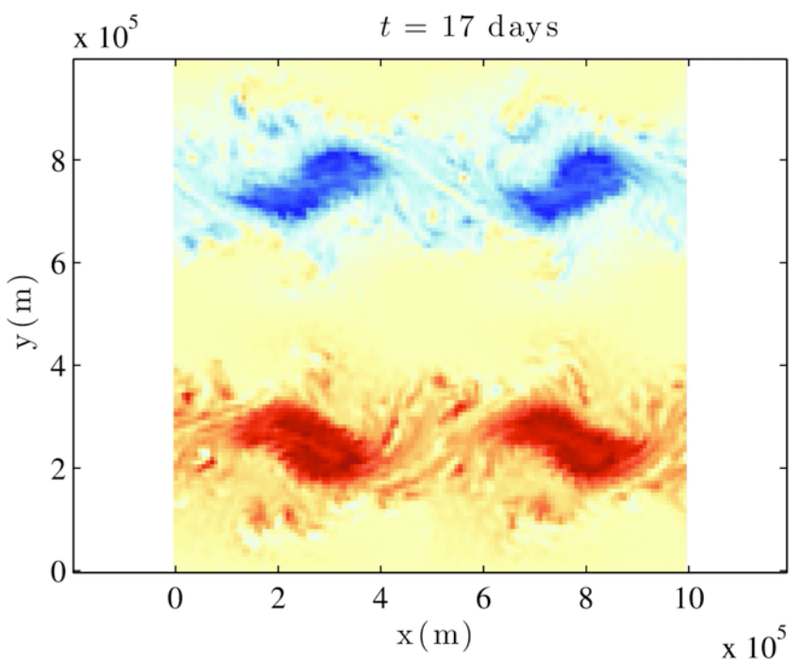
Stochastic 128x128

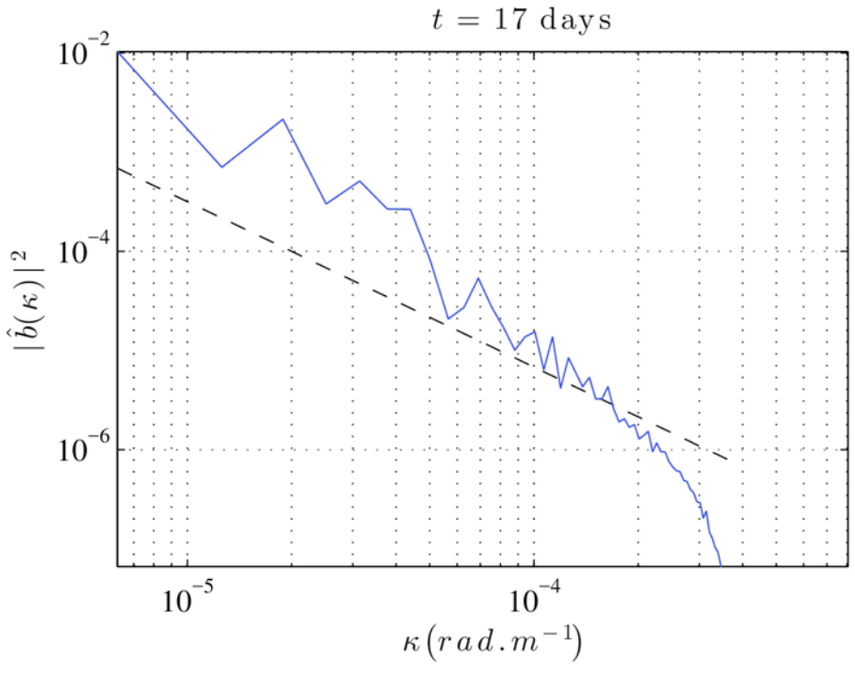


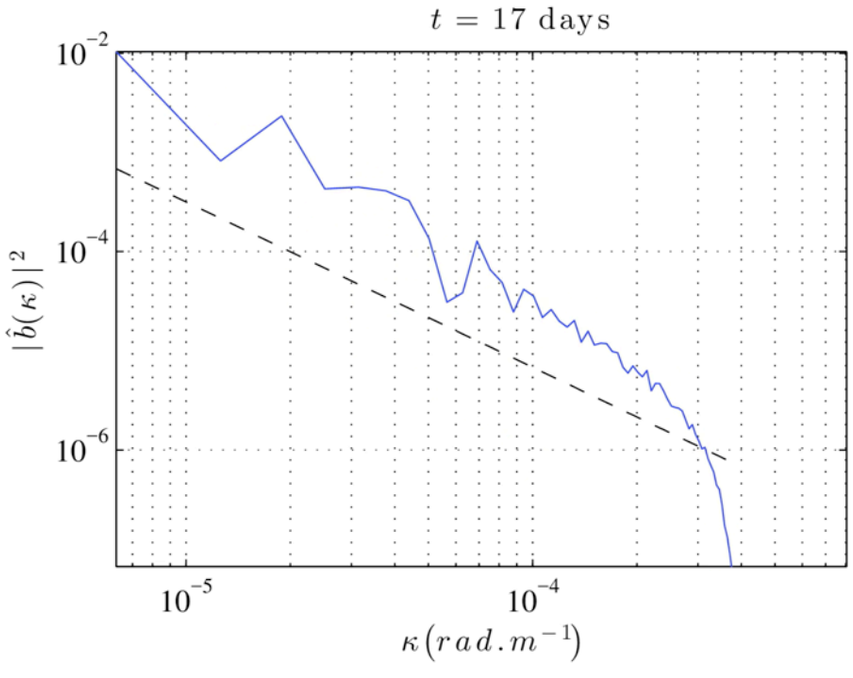
One realization

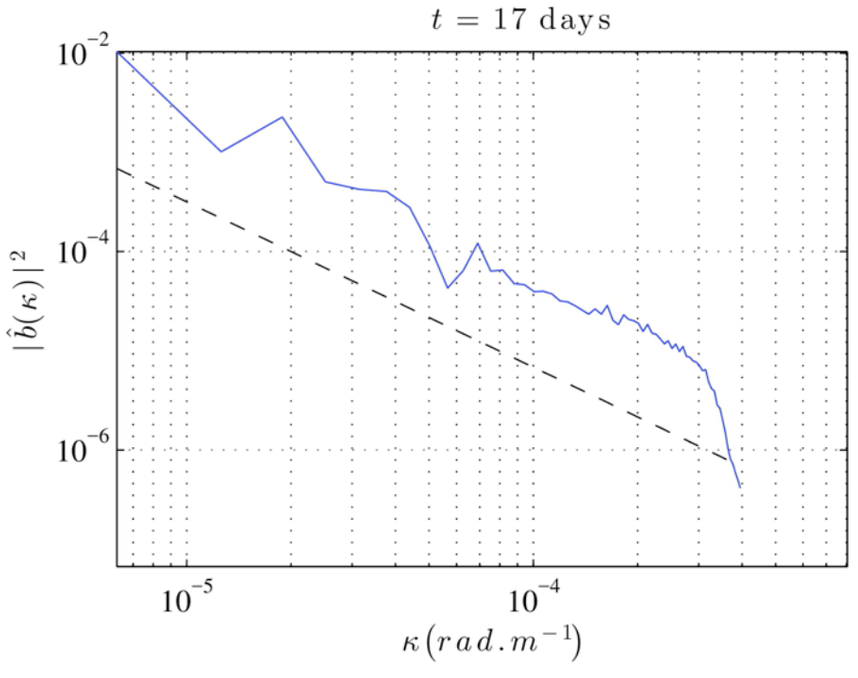




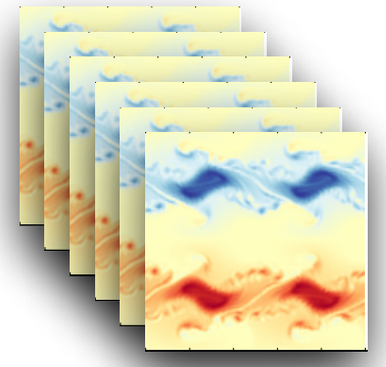




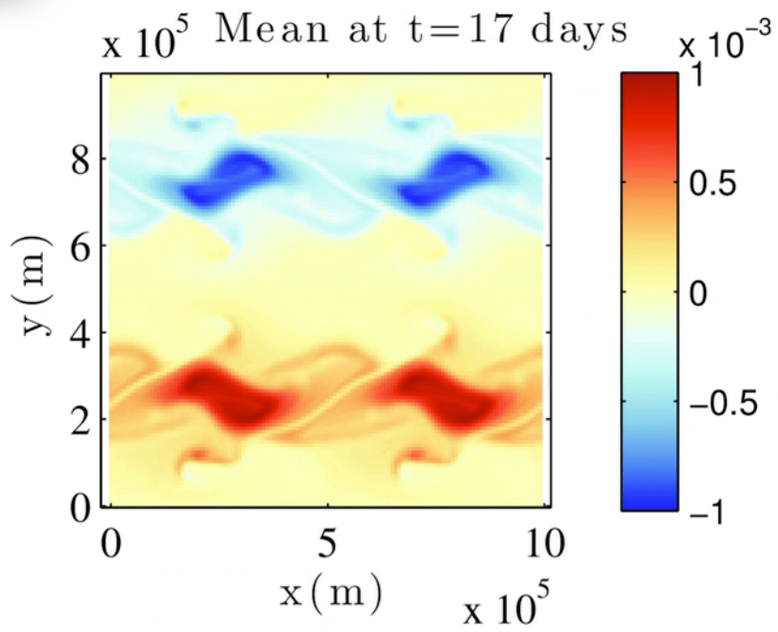


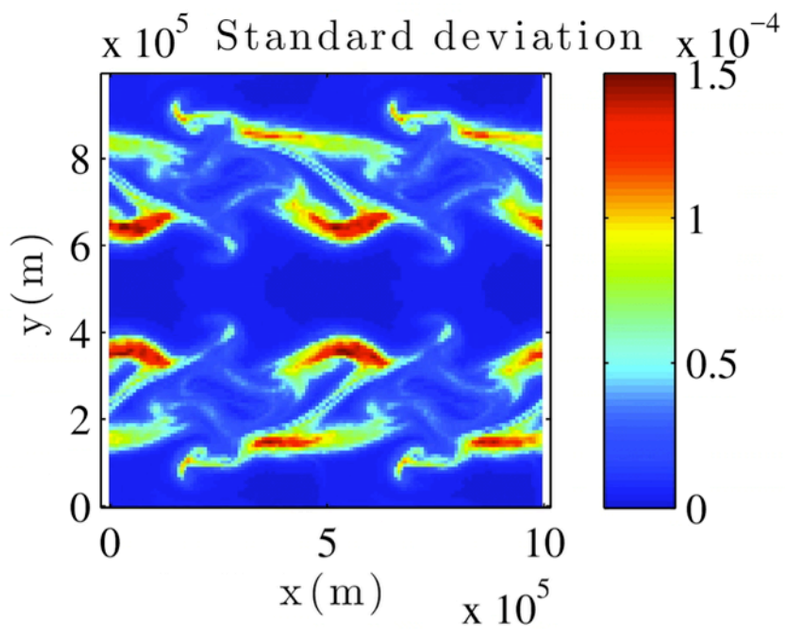


Lower noise **Our model** **Larger noise**



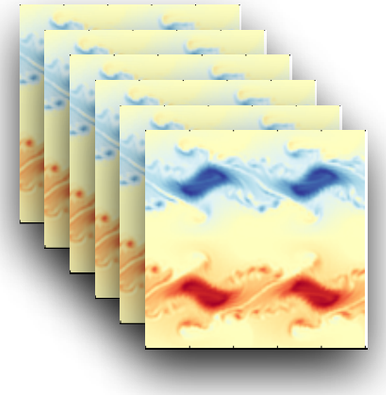
Ensemble


x 10^5 Mean at t=17 days
x 10^-3
y (m)
x (m)
x 10^5

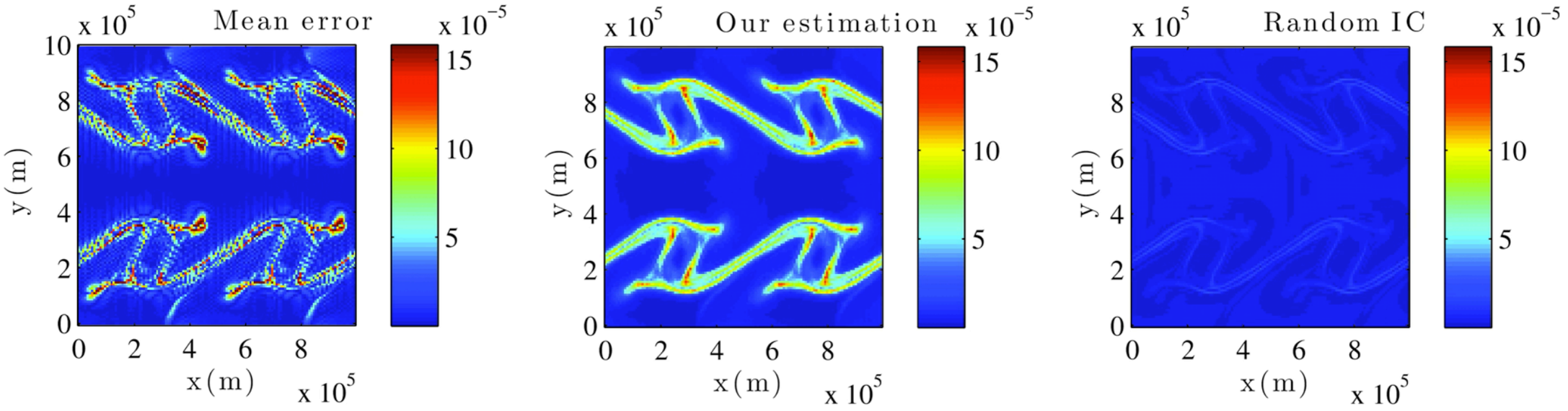

x 10^5 Standard deviation
x 10^-4
y (m)
x (m)
x 10^5

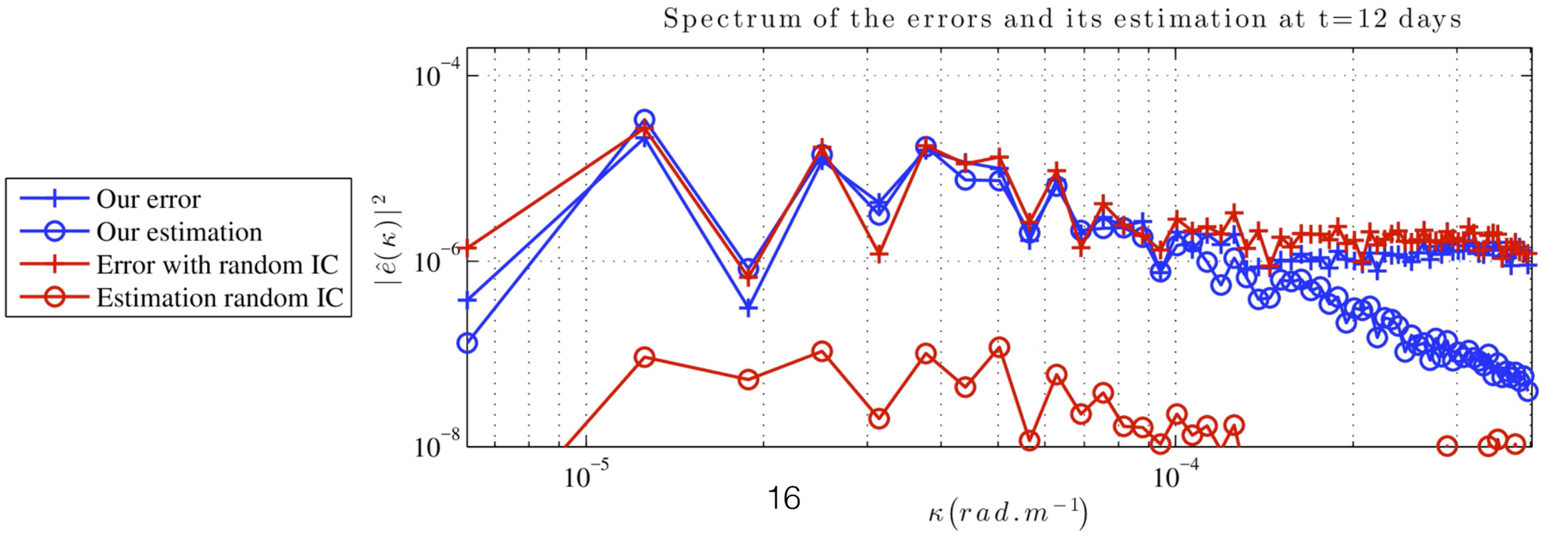
x 10^5 Skewness
y (m)
x (m)
x 10^5

x 10^5 log(Kurtosis-3)
y (m)
x (m)
x 10^5



Ensemble


Mean error
Our estimation
Random IC
x 10^5
x 10^-5
y (m)
x (m)
0
2
4
6
8
10
5
10
15


Spectrum of the errors and its estimation at t=12 days
Our error
Our estimation
Error with random IC
Estimation random IC
$|\hat{e}(\kappa)|^2$
$\kappa(rad.m^{-1})$
10^{-4}
10^{-6}
10^{-8}
10^{-5}
10^{-4}

Conclusion

Conclusion

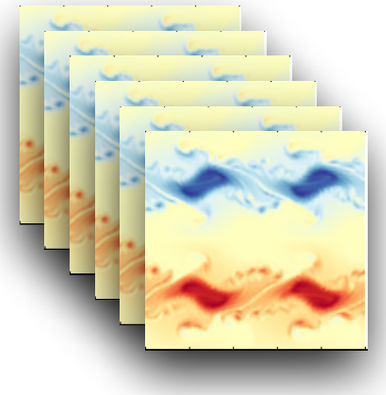
- Random transport applicable to any dynamics
- Better small scales
- Estimate position and amplitude of errors
- Extreme events
- Likely scenarios
- under Strong Uncertainty:
  Simple 2D description of frontolysis/frontogenesis

Thank you for your attention

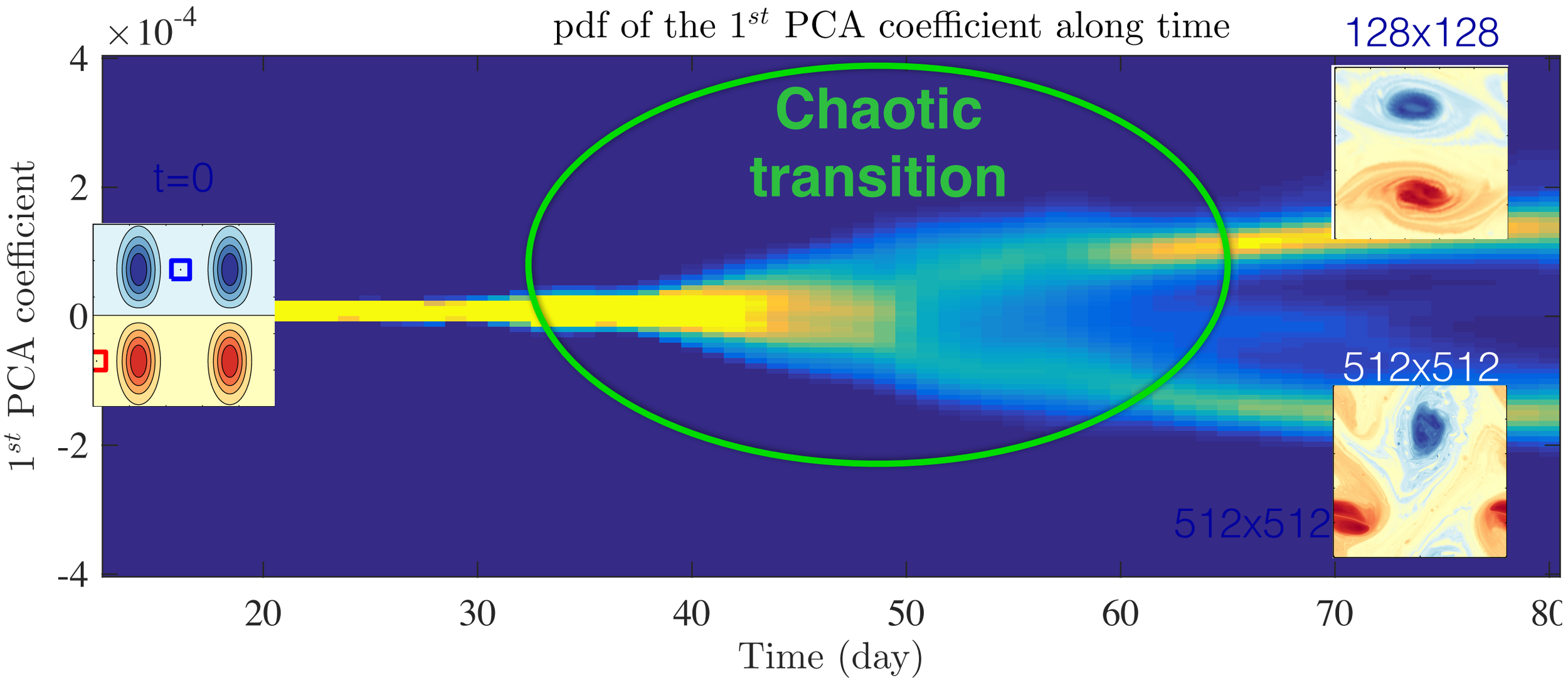
Code SQG MU:
link from Fluminance website - V. Resseguier

Likely SQG scenarios

tracked by SQG MU



Ensemble


t=70 days
pdf of the 1st PCA coefficient along time
×10-4
128x128
Chaotic
transition
t=0
512x512
512x512
1st PCA coefficient
Time (day)
4
2
0
-2
-4
20
30
40
50
60
70
80

SQG under Strong Uncertainty

SQG SU

Mesoscale divergence

Geostrophic balance

$$\boldsymbol{f} \times \boldsymbol{u} = -\frac{1}{\rho_b} \boldsymbol{\nabla} p'$$

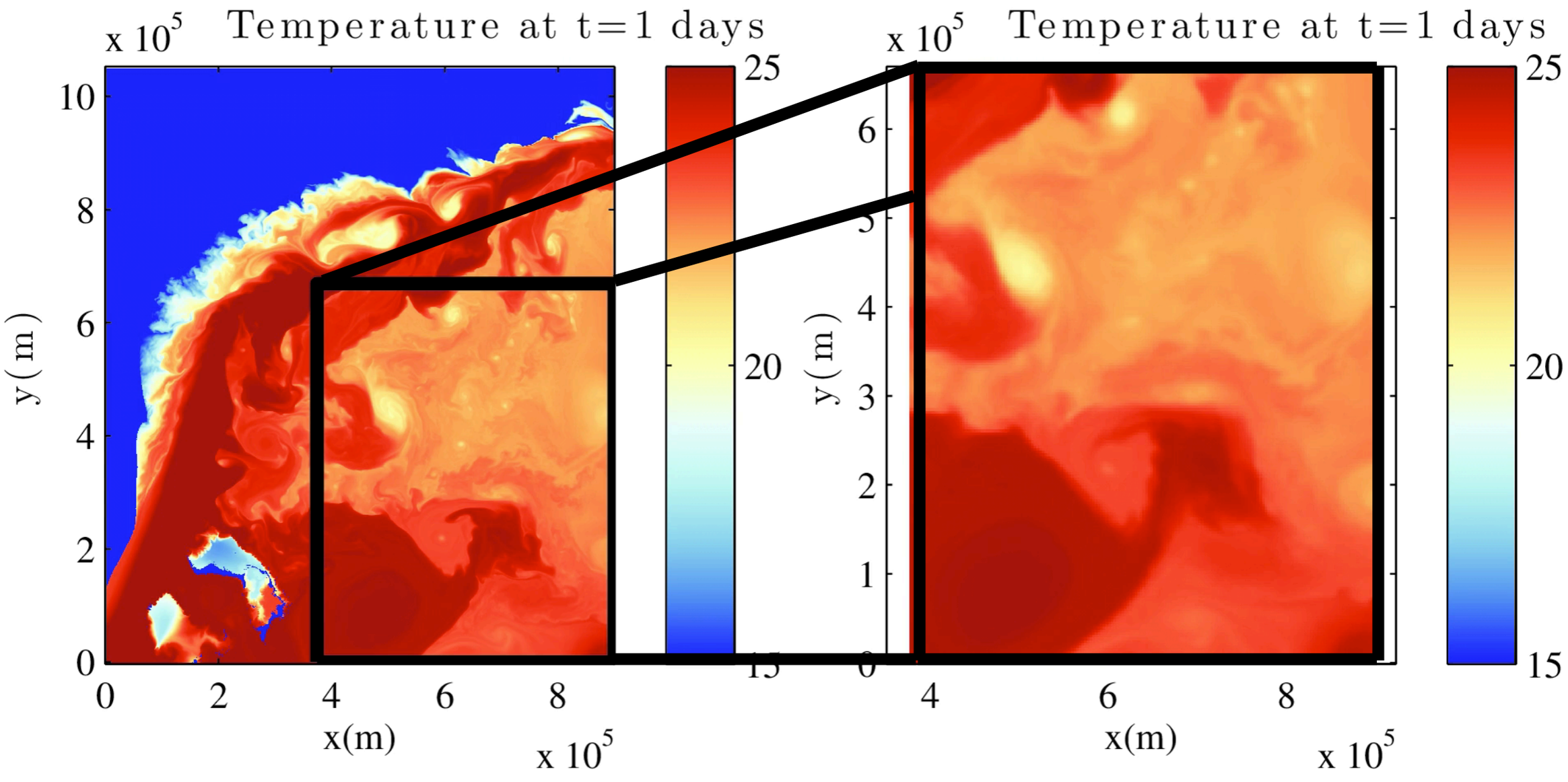
Horizontal Diffusion

$$+ \frac{a}{2} \Delta \boldsymbol{u}$$

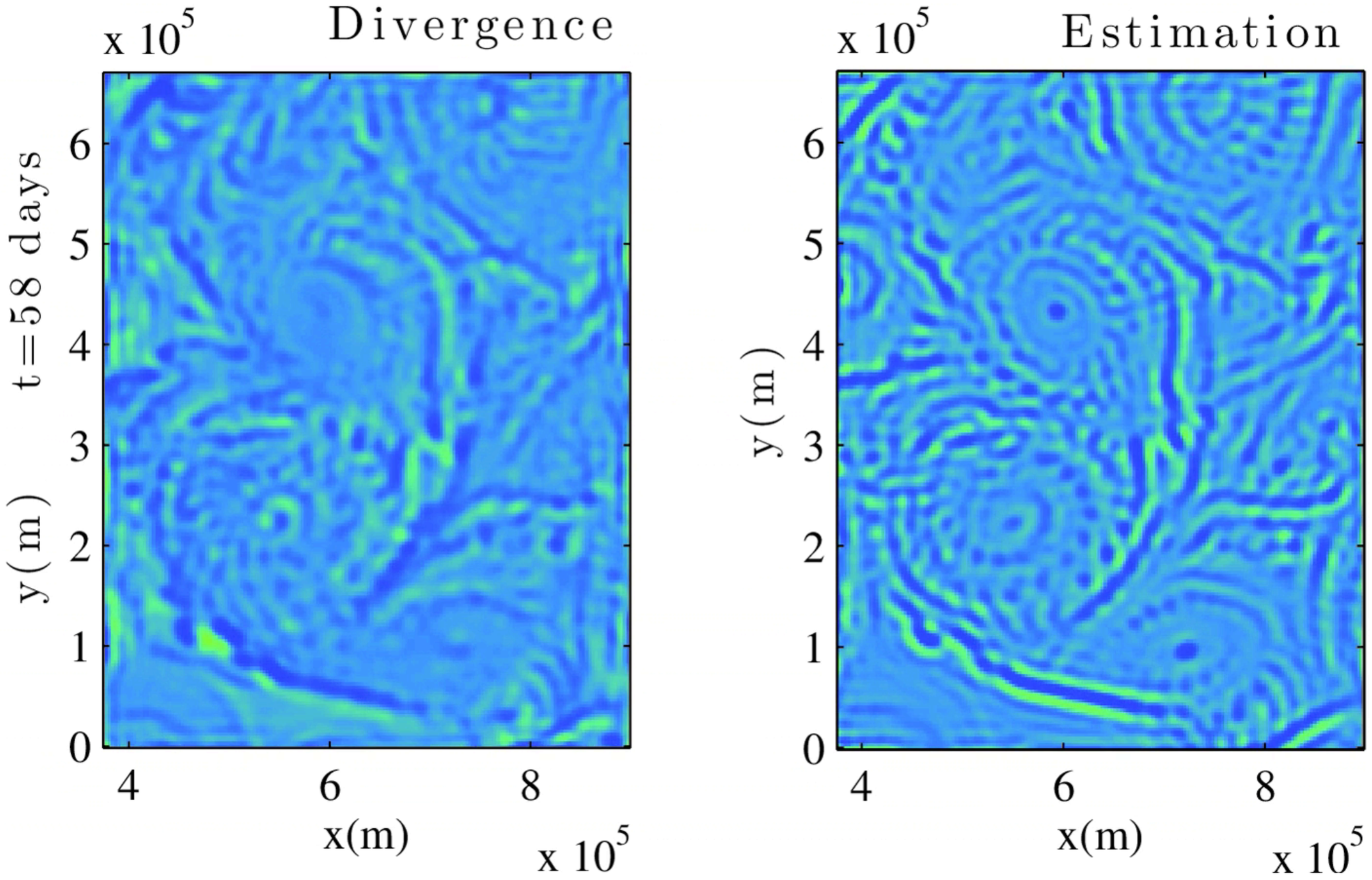
$$\boldsymbol{\nabla} \cdot \boldsymbol{u} \propto \Delta \boldsymbol{\nabla}^{\perp} \cdot \boldsymbol{u}$$

Filtering of model outputs:

Gula, Jonathan, M. Jeroen Molemaker, and James C. McWilliams
"Gulf Stream dynamics along the southeastern US seaboard."
Journal of Physical Oceanography 45.3 (2015): 690-715.



Spatial test



Spectral test